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Scale Invariant Follmann-type Tests

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Abstract: Suppose $X_1, X_2, ..., X_n$ is a random sample from the $N_p(\theta, V)$ distribution. Consider H_0: $\theta_1 = \theta_2 = ... = \theta_p = 0$ and H_1: $\theta_i \geq 0$ for i = 1, 2, ..., p, let H_1-H_0 denote the hypothesis that H_1 holds but H_0 does not and let~H_0 denote the hypothesis that H_0 does not hold. Because the Likelihood Ratio Test (LRT) of H_0 versus H_1-H_0 is complicated, several ad hoc tests have been proposed. The proposed test is a permutation and scale invariant test statistic which includes information about the correlation structure in the sum of the sample mean. The simulation study showed that it maintain type I error rate level very well and it also give good powers. The proposed test also is compared with the existing one with these invariance properties.

Key words: Follmann's test, modified Follmann's test, one-sided likelihood ratio tests, Tang-Gnecco-Geller test

INTRODUCTION

Consider a matched-pair design with p-dimensional responses. With $\theta = (\theta_1, \theta_2, ..., \theta_p)'$ the difference, treatment one minus treatment two, of the mean responses, one may test the null hypothesis, H_0: $\theta_1 = \theta_2 = ... = \theta_p = 0$, to determine if there is a significant difference in the two treatments. If one believes that, for each coordinate, the mean response for treatment one is at least as large as the mean response for treatment two, then the alternative can be constrained by H_1: $\theta_i \geq 0$ for i = 1, 2, ..., p. Follmann (1996) discussed other situations in which these order-restricted hypotheses are of interest.

Let H_1-H_0 denote the hypothesis that H_1 holds but H_0 does not and let $\sim H_0$ denote the hypothesis that H_0 does not hold. Let $X_1, X_2, ..., X_n$ be a random sample from the p-dimensional multivariate normal distribution with unknown mean $\theta = (\theta_1, \theta_2, ..., \theta_p)'$ and positive definite covariance matrix V. The sample mean and unbiased sample covariance are:

$$\bar{X} = \sum_{j=1}^{n} X_j / n \text{ and } \hat{S} = \sum_{j=1}^{n} (X_j - \bar{X})(X_j - \bar{X})'/(n-1)$$

It is well known that $\hat{S}$ is positive definite with probability one for n>p. Kudo (1963), Shorack (1967) and Perlman (1969) derive the Likelihood Ratio Test (LRT) of H_0 versus H_1-H_0 if V is known, known up to a multiplicative constant or completely unknown, respectively. By V known up to a multiplicative constant, we mean $V = \sigma^2 V_0$ with V_0 known and σ unknown.

Tang *et al.* (1989) proposed approximate LRTs and Follmann (1996) studied one-sided modifications of the non-directional χ^2 and Hotelling's T^2 tests of H_0 versus $\sim H_0$. Follmann's tests reject H_0 if the appropriate non-directional ones do with significance level 2α and:

$$\bar{X}_1 + \bar{X}_2 + ... + \bar{X}_p > 0 \qquad (1)$$

The tests that use (Eq. 1) or a variant of it are called Follmann-type tests and they include those in Chongcharoen *et al.* (2002) which incorporate information about the off-diagonal elements of V in Eq. 1. The latter kind of Follmann-type tests is called the new tests. All three of these procedures, approximate LRTs, Follmann's tests and new tests, are easier to implement than the LRTs but the two Follmann-type tests are easier to use than the approximate LRT. In particular, the Follmann-type tests utilize chi-square or F critical values but the null distributions of the approximate LRT statistics are mixtures of chi-square or beta distributions.

It is clear that for most matched-pair designs, one wants the test to be invariant under changes in the units of measurement for any or all of the response variables as well as changes in the order of the response variables. The likelihood function and the constraint region, H_1, are invariant under permutations of the indices of the response variables and under scale changes for the

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